

Light Cone Reflection and the Spectrum of Neutrinos

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Abstract

We extend the treatment of neutrinos within the context of SIM(2) Very Special Relativity (VSR) by adding a new discrete symmetry that we call Light Cone Reflection (LCR). We construct a Lagrangian that exhibits both VSR and LCR symmetry, and find that the spectrum involves two neutrinos, one tachyonic and the other not, with the same absolute value of the mass parameter. We argue that LCR symmetry offers a rationale for introducing tachyonic neutrinos.

Introduction

During the recent flurry of interest in tachyonic neutrinos occasioned by the rise[1] and fall[2] of the result from the OPERA experiment, it became clear that although the idea of their existence had been proposed long ago[3, 4], there was no compelling theoretical reason behind it. Contrariwise, there were powerful theoretical arguments, most notably those in reference [5], that seriously questioned whether OPERA had even seen what it claimed to see.

Notwithstanding the climate of doubt surrounding the OPERA result, it is still possible that at least one of the neutrino mass eigenstates is tachyonic, although the effect in a time-of-flight measurement will necessarily be less than the parts per 10^5 that OPERA reported. In anticipation of future experiments, possibly with renewed claims of a positive signal, one of the motivations of this paper is to offer a theoretical rationale for the existence of tachyonic neutrinos.

In their 1985 paper, the authors of ref. [4] proposed the equation

$$(i\not{\nabla}\gamma^5 - m)\psi = 0$$

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to describe tachyonic neutrinos, in the hope that a fully Lorentz-invariant theory could be constructed. However, a quantum theory based on this equation led to difficulties that have not been fully overcome. Nor did the theory gain any new symmetry or other desirable features as a result of the properties of this equation.

Meanwhile, the idea that Lorentz symmetry might be broken began to gain some traction[6], and, in particular, Colladay and Kostelecký introduced[7] the Standard Model Extension, which systematized the search for Lorentz-violating effects. Recently, Kostelecký and Mewes[8] used the SME to provide a detailed discussion of Lorentz violating effects in the neutrino sector.

In 2006, Cohen and Glashow[9] introduced a form of Lorentz violation, called Very Special Relativity (VSR), which proposes that the relativistic symmetry is one of four particular subgroups of the Lorentz Group. CG argue that almost all the well-known consequences of Lorentz invariance follow from VSR as well. In addition, VSR requires that the P, T and CP symmetries all be broken. Imposing any one of them elevates the symmetry to the full Lorentz Group.

In a second paper[10], CG concentrate on the largest of the four subgroups, the maximal subgroup SIM(2), which is a four parameter group. They study neutrinos in this context, pointing out that it is possible to have neutrino masses that neither violate lepton number nor require the existence of sterile neutrinos, and positing a non-local wave equation for the neutrino.

To implement the SIM(2) version of VSR, one introduces a null four-vector, n^μ , such that the physics can depend on the spatial direction of n but not its magnitude. If we label this direction to be the z axis, then n^μ has the form $n^\mu = a(1, 0, 0, 1)$, where a is an arbitrary non-zero real number on which the physics cannot depend. We shall therefore set $a = 1$.

It is the purpose of this paper to point out that, although VSR reduces the symmetry compared to normal Special Relativity, the existence of n^μ allows us to enlarge the symmetry in a different way, by introducing a new, discrete, spacetime symmetry that lies outside the Lorentz Group. We call this symmetry Light Cone Reflection, or LCR.

In the following sections, we define LCR, derive some of its properties, and relate it to a new one-parameter family of spacetime transformations. Then, building on a Lagrangian introduced by Álvarez and Vidal[11], we construct a theory of neutrinos that is both VSR and LCR invariant, and discuss some of its features.

LCR and its Continuous Generalization

Consider the transformation

$$x^\mu \rightarrow y^\mu = x^\mu - n^\mu \frac{x^2}{n \cdot x}$$

where n^μ is the null vector associated with the SIM(2) symmetry. Note that the transformation is independent of the overall scale of n^μ , as required. We call this “Light Cone Reflection”, principally because

$$y^2 = -x^2$$

i.e. LCR maps timelike vectors into spacelike ones, and vice-versa. Also, we see that

$$n \cdot y = n \cdot x$$

and

$$x^\mu = y^\mu - n^\mu \frac{y^2}{n \cdot y}$$

i.e if we repeat the transformation we get back where we started. In addition,

$$x \cdot y = 0$$

This transformation is defined for all x such that $n \cdot x \neq 0$. Since n is null, $n \cdot x$ cannot vanish for any timelike x , and for lightlike x , it vanishes only for x proportional to n . There is, however, a subset of spacelike x for which $n \cdot x$ vanishes. In some sense, there are “more” spacelike than timelike vectors, so an invertible mapping between the two kinds of vectors cannot include all the spacelike vectors.

A simple calculation yields

$$\frac{\partial}{\partial y^\nu} = \frac{\partial}{\partial x^\nu} - \frac{\partial}{\partial x^\nu} \left[\frac{x^2}{n \cdot x} \right] n^\mu \frac{\partial}{\partial x^\mu}.$$

Observing that

$$n \cdot \frac{\partial}{\partial x} \left[\frac{x^2}{n \cdot x} \right] = 2,$$

we see that

$$n \cdot \frac{\partial}{\partial y} = -n \cdot \frac{\partial}{\partial x}.$$

This property will play a key role in constructing a VSR- and LCR-invariant theory in the next section.

We close this section with a brief discussion of a one-parameter group of transformations that includes LCR. It is convenient to write the transformation in the form

$$y^\mu(\alpha) = x^\mu - \frac{1}{2}(1 - \alpha) n^\mu \frac{x^2}{n \cdot x}$$

Written this way, the product of a transformation parameterized by α and one parameterized by β is simply given by a transformation parameterized by the

product $\alpha\beta$. The identity is $\alpha = 1$, LCR is $\alpha = -1$, and the value $\alpha = 0$ must be excluded because it doesn't have an inverse.

The earlier formulas generalize easily to accommodate α . We have:

$$y^2 = \alpha x^2;$$

$$n \cdot y = n \cdot x;$$

$$x^\mu = y^\mu - \frac{1}{2} \left(1 - \frac{1}{\alpha} \right) n^\mu \frac{y^2}{n \cdot y};$$

$$x \cdot y = \frac{1}{2} (1 + \alpha) x^2;$$

$$\frac{\partial}{\partial y^\nu} = \frac{\partial}{\partial x^\nu} - \frac{1}{2} \left(1 - \frac{1}{\alpha} \right) \frac{\partial}{\partial x^\nu} \left[\frac{x^2}{n \cdot x} \right] n^\mu \frac{\partial}{\partial x^\mu};$$

and

$$n \cdot \frac{\partial}{\partial y} = \frac{1}{\alpha} n \cdot \frac{\partial}{\partial x}.$$

Finally, we observe that

$$\frac{\partial y^\mu}{\partial x^\nu} = \delta_\nu^\mu - \frac{1}{2} (1 - \alpha) n^\mu \frac{\partial}{\partial x^\nu} \left[\frac{x^2}{n \cdot x} \right],$$

and, with $n^\mu = (1, 0, 0, 1)$, we can explicitly evaluate the Jacobean of the transformation,

$$\left| \frac{\partial y^\mu}{\partial x^\nu} \right| = |\alpha|.$$

VSR- and LCR-invariant Lagrangian

Cohen and Glashow postulate the following VSR-invariant non-local wave equation for the neutrino:

$$\left(\not{p} - \frac{1}{2} \frac{m^2}{n \cdot p} \not{n} \right) \nu = 0$$

which implies the dispersion formula $p^2 = m^2$.

Álvarez and Vidal [11] exhibited a Lagrangian density that yields the CG equation:

$$L = i \bar{\nu} \not{\partial} \nu + i \bar{\chi} n \cdot \partial \psi + i \bar{\psi} n \cdot \partial \chi + \frac{i}{2} m [\bar{\chi} \not{n} \nu + \bar{\psi} \nu - \bar{\nu} \not{n} \chi - \bar{\nu} \psi]$$

When n^μ is scaled by a VSR transformation, χ must be scaled oppositely to compensate.

Let us generalize the Lagrangian slightly:

$$L = i\bar{\nu}\not{\partial}\nu + i\bar{\chi}n\cdot\partial\psi + i\bar{\psi}n\cdot\partial\chi + \frac{i}{2}[m_1\bar{\chi}\not{n}\nu + m_2\bar{\psi}\nu - m_1^*\bar{\nu}\not{n}\chi - m_2^*\bar{\nu}\psi]$$

This still yields the CG equation, but now with m^2 replaced by $\text{Re}(m_2^*m_1)$.

When we make an LCR transformation, we have two things to worry about:

(i) the $\bar{\nu}\not{\partial}\nu$ term is not invariant, because

$$\partial_\mu \rightarrow \partial_\mu - \partial_\mu \left[\frac{x^2}{n\cdot x} \right] n\cdot\partial$$

and (ii) the terms $i\bar{\chi}n\cdot\partial\psi + i\bar{\psi}n\cdot\partial\chi$ change sign because $n\cdot\partial \rightarrow -n\cdot\partial$.

We consider the second problem first. We can compensate for the sign change by changing the sign of either χ or ψ (but not both). Let us choose ψ . Then the mass terms proportional to m_2 and m_2^* change sign. To fix this, we relabel ν as ν_1 and introduce a second neutrino field ν_2 , with opposite sign of the m_2 mass terms compared to ν_1 . The Lagrangian then becomes

$$\begin{aligned} L = & i\bar{\nu}_1\not{\partial}\nu_1 + i\bar{\nu}_2\not{\partial}\nu_2 + i\bar{\chi}n\cdot\partial\psi + i\bar{\psi}n\cdot\partial\chi + \frac{i}{2}[m_1\bar{\chi}\not{n}\nu_1 + m_2\bar{\psi}\nu_1 - m_1^*\bar{\nu}_1\not{n}\chi - m_2^*\bar{\nu}_1\psi \\ & + m_1\bar{\chi}\not{n}\nu_2 - m_2\bar{\psi}\nu_2 - m_1^*\bar{\nu}_2\not{n}\chi + m_2^*\bar{\nu}_2\psi] \end{aligned}$$

Under LCR, $n\cdot\partial \rightarrow -n\cdot\partial$, $\chi \rightarrow \chi$, $\psi \rightarrow -\psi$, and $\nu_1 \longleftrightarrow \nu_2$, leaving L invariant (except for the $\not{\partial}$ terms, which we will treat below).

Variation of this Lagrangian yields coupled equations for ν_1 and ν_2 which, when diagonalized, lead to a pair of CG equations, one with m^2 replaced by $|m_1m_2|$ and the other with m^2 replaced by $-|m_1m_2|$; i.e. the spectrum contains a pair of neutrinos, one a normal particle, and the other a tachyon.

To deal with the non-invariance of $\bar{\nu}\not{\partial}\nu$, we introduce a vector field A_μ by replacing the derivative ∂_μ in $\not{\partial}$ with a covariant derivative D_μ :

$$\partial_\mu \rightarrow \partial_\mu - \frac{1}{M}A_\mu n\cdot\partial \equiv D_\mu.$$

(Although we call the new field A_μ , we do not mean to suggest that it is related to the photon.) We need to choose the transformation of A_μ so that D_μ is invariant under LCR. Let $V(x) \equiv \frac{x^2}{n\cdot x}$. Then $n\cdot\partial V = 2$. We have

$$\partial'_\mu = \partial_\mu - \partial_\mu V n\cdot\partial$$

and

$$n\cdot\partial' = -n\cdot\partial$$

so

$$D'_\mu = \partial_\mu - \partial_\mu V n \cdot \partial + \frac{1}{M} A'_\mu n \cdot \partial.$$

Therefore, with

$$A'_\mu = -A_\mu + M \partial_\mu V$$

we have $D'_\mu = D_\mu$, as required. The appearance of the dimensionful parameter M in the coupling of A_μ means that the interaction is non-renormalizable, and suggests that this Lagrangian will give way to new physics at energy scales $\sim M$.

In addition to replacing ∂_μ with D_μ , for each ν_i we must also add the term

$$\frac{-i}{2M} \bar{\nu} \gamma^\mu \nu n \cdot \partial A_\mu$$

to \mathcal{L} , in order to preserve its Hermiticity.

Having introduced A_μ , we need to find a suitable kinetic term that is both VSR and LCR invariant. This turns out to be a little messy. Standard operating procedure dictates that we evaluate

$$[D_\mu, D_\nu] = -\frac{1}{M} F_{\mu\nu} n \cdot \partial,$$

which yields

$$F = \partial_\mu A_\nu - \partial_\nu A_\mu - \frac{1}{M} (A_\mu n \cdot \partial A_\nu - A_\nu n \cdot \partial A_\mu).$$

It is easily verified that, under LCR, $F_{\mu\nu} \rightarrow -F_{\mu\nu}$. One cannot, however, simply use $-\frac{1}{4} F_{\mu\nu} F^{\mu\nu}$ as the kinetic term for A_μ . We see from the definition of D_μ that scaling n^μ under VSR must be accompanied by an inverse scaling of A_μ . Therefore $-\frac{1}{4} F_{\mu\nu} F^{\mu\nu}$ is not VSR invariant. Each factor of F must be accompanied by an additional factor of n . We define

$$J_\nu = n^\mu F_{\mu\nu}.$$

Then $J^\mu J_\mu$ has the necessary symmetry. However, let us define the additional quantities

$$Q_\nu = n \cdot \partial A_\nu,$$

and

$$K_\nu = \partial_\nu (n \cdot A) - \frac{1}{M} A_\nu n \cdot \partial (n \cdot A).$$

We observe that, under LCR, $Q_\nu \rightarrow Q_\nu$, and $K_\nu \rightarrow -K_\nu$, and

$$J_\nu = Q_\nu - K_\nu - \frac{1}{M} (n \cdot A) Q_\nu.$$

Thus from invariance requirements alone, a suitable kinetic term would be $a_1 J_\nu J^\nu + a_2 K_\nu K^\nu + a_3 Q_\nu Q^\nu$, where the a_i are arbitrary coefficients. It might be possible to impose the additional LCR-invariant constraint $Q_\nu = 0$, in which case, we would have $J_\nu = -K_\nu$, and the remaining kinetic term would be essentially unique.

Conclusions

This paper is being written at a moment when the idea of tachyonic neutrinos, never one of great popularity, is at a particularly low ebb. Despite interesting theoretical and phenomenological contributions over the years, for example references [12, 13, 14, 15, 16, 17, 18], this has always been the pursuit of a small minority, and now the majority has been convinced, by the retraction of the OPERA result and the report of some new experiments[2], that neutrinos have been shown not to travel faster than light.

Of course this is not what has been shown. OPERA claimed, in a time of flight measurement of neutrinos whose energies averaged 17 GeV, that the speed exceeded that of light by some parts in 10^5 . This is an extremely awkward result, because, assuming that the superluminal neutrinos obey a Lorentz-invariant dispersion formula, the mass parameter has to be in the range of hundreds of MeV. The demise of this result is not cause for concern among aficionados of the tachyonic neutrino hypothesis. Assuming that the neutrinos have mass parameters in the meV range, the deviation from light speed in the OPERA experiment, or in any of the other time of flight measurements so far reported, would be unmeasurably small.

Theoretical papers on tachyonic neutrinos, and on tachyons more generally, have mainly been concerned with the question of whether a sensible theory involving superluminal particles can be formulated. The difficulties in this regard are quite formidable, and despite a number of careful and ingenious approaches, there is so far no generally accepted solution. The purpose of this paper is somewhat different. We want to make the argument not that tachyons are possible, but that, if introduced appropriately, they can be viewed as the necessary concomitant of a new symmetry of nature. We have exhibited such a symmetry, constructed an invariant Lagrangian, and shown that tachyons and non-tachyons arise symmetrically in its spectrum.

At best our Lagrangian is only an effective one; the derivative couplings that we have introduced probably mean that it is non-renormalizable. Many open questions remain, among them whether an LCR-invariant theory can be successfully quantized, whether it can be incorporated into an extension of the Standard Model, and whether it can be extended to the continuous symmetry that we discussed briefly above. An obstacle to the latter is the factor of $|\alpha|$ contributed by the Jacobean, compensating for which probably requires the addition of at least one more field.

Finally we note that, in VSR alone, one could contemplate restricting explicit deviations from full Lorentz symmetry to the neutrino sector, with leakage to the rest of the Standard Model being quite small. In our extension to LCR, we have introduced the gauge-like field A_μ , which must accompany all derivatives (except those in the form $n \cdot \partial$) in the full Lagrangian in order to maintain the symmetry. The effects of A_μ are suppressed by the factor $\frac{1}{M}$, but the consequences of imposing LCR will inevitably be more widespread than those of VSR alone.

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